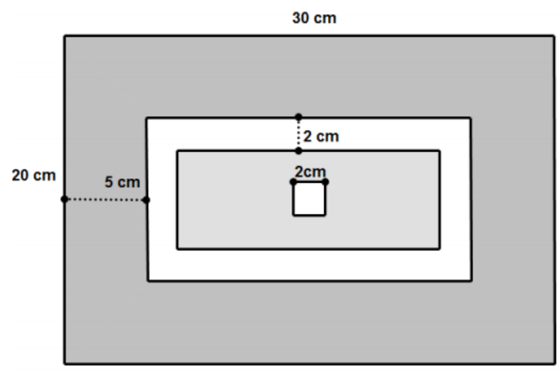
1) Jazmin has a coin, and Sue has a standard 6-sided die. Jazmin flips the coin, and Sue rolls the die. What is the probability that Jazmin will flip a head *and* Sue will roll an even number?

2) In the Monopoly board game, a player tosses two six-sided dice and adds the two numbers to determine how many spaces to move. Albert wants to play Monopoly but can't find the dice. He finds a 12-sided die with the numbers 1 through 12 on it to replace the two regular 6-sided dice. What is the ratio of the probability of rolling a 7 with the two 6-sided dice to the probability of rolling a 7 with the 12-sided die? Express your answer in the format "whole number : whole number".

3) Ted spends some afternoons throwing darts. The length of the board is 30 cm and the width is 20 cm as shown in the scale drawing. The outer grey band has a width of 5 cm on each side, the next white band has a width of 2 cm on each side and the inner white square has side length 2 cm. His darts always hit the board, but assume at random. What is the probability he will hit the inner light grey band? Express your answer as a reduced fraction (i.e. in lowest terms) or as a decimal rounded to the hundredths place.

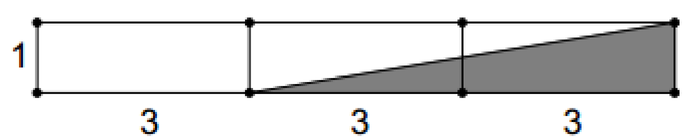


4) A factor of 60 is chosen at random. What is the probability that this factor has factors of 2 and 5? Express your answer as a fraction.

5) Katie and Jim play a game with 2 six sided number cubes numbered 1 through 6. When the number cubes are rolled, Katie gets a point if the sum of the two is even and Jim gets a point if the product is even. What is the likelihood that on one roll of both cubes both Katie and Jim get a point?

6) Mr. Zee just can't play it straight. He's asked you to come over and paint his fence. He'll pay you based on the flip of 3 coins. If they all come up heads you get $100. If two heads, then $75, if one head $50 and no heads $25. What's the likelihood you will earn more than $50 for this job?

7) A dart hits the dartboard shown at random. Find the probability of the dart landing in the shaded region.



8) How many times should a coin be tossed to have at least a 95% chance of it landing "heads" up at least once?

BONUS PROBLEMS

1. An animal cage is holding 7 black cats and 5 white cats. None of them want to be in there. The cage door is opened slightly and two cats escape. What is the probability that the escaping cats are both white?

2. The names of 10 boys and 11 girls from your class are put into a hat. What is the probability that the first two names chosen will be a boy followed by a girl?

3. What is the probability of rolling an even number with a fair six-sided die followed by flipping heads twice in a row with a fair coin?

4. A drawer contains 18 socks, some black and some white. Two socks are randomly drawn from the drawer. The probability that both socks are black is 26/51. How many socks in the drawer are white?

Solutions

1) The probability of flipping a head on the coin is 1/2. The probability of rolling an even number on the die is also 1/2. The probability of both events occurring is the product of their individual probabilities, or ½ × ½.
Answer: 1/4

2) Using the two six-sided dice, here are all the ways in which you can end up with a sum of 7:

    1 + 6
    6 + 1
    2 + 5
    5 + 2
    3 + 4
    4 + 3

    So, there are six ways you can get a sum of 7, out of a total of 36 possibilities. So, the probability of getting a 7 is 6/36, or 1/6.

    With the 12-sided die, there is just one way to get a 7, and there are 12 possibilities. So, the probability is 1/12.

    Now, we have to put these two probabilities together to make a ratio:

$$\frac{1}{6} : \frac{1}{12}$$

    Multiplying both sides by 12, we get:

$$2 : 1$$

    Answer: 2 : 1

3) We need to compare the area of the light gray band to the area of the entire board. The light gray band is a rectangle, minus the small 2x2 square in the middle. The dimensions of the rectangle are 16 cm (length) x 6 cm (height). So, the area of the light gray band is:

$$(16\text{ cm} \times 6\text{ cm}) - (2\text{ cm} \times 2\text{ cm}) = 92\text{ cm}^2$$

    The area of the entire board is:

$$30\text{ cm} \times 20\text{ cm} = 600\text{ cm}^2$$

So, the probability of hitting the light gray band is:

$$\frac{92}{600} = \frac{46}{300} = \frac{23}{150}$$

Answer: 23/150

4) The factors of 60 are:

1, 2, 3, 4, 5, 6, **10**, 12, 15, **20**, **30**, **60**

There are 12 factors. Of these, four of them are divisible by 2 and 5. They are 10, 20, 30, and 60. So the probability is 4/12 or 1/3.

Answer: 1/3

5) Katie gets a point if the sum is even. This happens when you roll either two odd numbers or two even numbers. Jim gets a point if the product is even. This happens when you have at least one even number:

First die	Second die	Sum	Product	Katie gets a point?	Jim gets a point?
1	1	2	1	Yes!	
1	2	3	2		Yes!
1	3	4	3	Yes!	
1	4	5	4		Yes!
1	5	6	5	Yes!	
1	6	7	6		Yes!
2	1	3	2		Yes!
2	**2**	**4**	**4**	**Yes!**	**Yes!**
2	3	5	6		Yes!
2	**4**	**6**	**8**	**Yes!**	**Yes!**
2	5	7	10		Yes!
2	**6**	**8**	**12**	**Yes!**	**Yes!**
3	1	4	3	Yes!	
3	2	5	6		Yes!
3	3	6	9	Yes!	
3	4	7	12		Yes!
3	5	8	15	Yes!	
3	6	9	18		Yes!
4	1	5	4		Yes!
4	**2**	**6**	**8**	**Yes!**	**Yes!**
4	3	7	12		Yes!
4	**4**	**8**	**16**	**Yes!**	**Yes!**
4	5	9	20		Yes!
4	**6**	**10**	**24**	**Yes!**	**Yes!**
5	1	6	5	Yes!	
5	2	7	10		Yes!

5	3	8	15	Yes!	
5	4	9	20		Yes!
5	5	10	25	Yes!	
5	6	11	30		Yes!
6	1	7	6		Yes!
6	**2**	**8**	**12**	**Yes!**	**Yes!**
6	3	9	18		Yes!
6	**4**	**10**	**24**	**Yes!**	**Yes!**
6	5	11	30		Yes!
6	**6**	**12**	**36**	**Yes!**	**Yes!**

The only time that both of them get a point is when you roll two even numbers, and this happens 9 out of 36 times, or ¼ of the time.

Answer: ¼

6) Here are all the possible outcomes when flipping three coins:

Flip 1	Flip 2	Flip 3	# of Heads	$ earnings
H	H	H	3	**$100**
H	H	T	2	**$75**
H	T	H	2	**$75**
H	T	T	1	$50
T	H	H	2	**$75**
T	H	T	1	$50
T	T	H	1	$50
T	T	T	0	$25

Exactly four out of the eight times, you will earn more than $50. So the probability is 4/8 or ½.

Answer: ½

7) The area of the shaded region, a triangle, is:

½ × base × height
½ × 6 × 1
3

The area of the entire dartboard, a rectangle is:

base × height
9 × 1
9

So, the probability of hitting the shaded region is 3/9, or 1/3.

Answer: 1/3

8) The probability of the opposite event is 100% - (probability of the original event). So the question can be rephrased as:

   How many times should a coin be tossed to have less than a 5% chance of landing tails up every time?

   With one coin toss, the probability of landing tails up every time = ½ = 50%
   With two coin tosses, the probability of landing tails up every time = ½ × ½ = 1/4 = 25%
   With three coin tosses, the probability of landing tails up every time = ½ × ½ × ½ = 1/8 = 12.5%
   With four coin tosses, the probability of landing tails up every time = ½ × ½ × ½ × ½ = 1/16 = 6.3%
   With five coin tosses, the probability of landing tails up every time = ½ × ½ × ½ × ½ × ½ = 1/32 = 3.1%

   Answer: 5 times

9) The first event that must occur is that one of the 5 white cats (out of 12) must escape. The probability of this is 5/12. Now there are only 11 cats left, and 4 of them are white. The probability of the 2nd cat being white is now 4/11. The probability of both events occurring is the product of the two probabilities.

   $$\frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$$

   Answer: 5/33

10) Initially, the probability of drawing a boy's name is 10/21. Once that has happened, there are 11 girls' names in the hat out of a total of 20 names remaining. So the probability of then drawing a girl's name is 10/20. The probability of both events occurring is the product of the two probabilities.

   $$\frac{10}{21} \times \frac{11}{20} = \frac{110}{420} = \frac{11}{42}$$

   Answer: 11/42

11) Individually, the probability of each of the described events occurring is ½. The probability of all three events occurring is ½ × ½ × ½.

   $$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

   Answer: 1/8

12) Using algebra, and the variable "b" to represent the number of black socks:

$\frac{b}{18} \cdot \frac{b-1}{17} = \frac{26}{51}$

$\frac{b(b-1)}{306} = \frac{26}{51}$

51b(b-1) = 7956
b(b-1) = 156
Since b must be 17 or less, and b-1 = the next lowest number, we try:
17 · 16 = 272
16 · 15 = 240
15 · 14 = 210
14 · 13 = 182
13 · 12 = 156 ← There must be 13 black socks

Answer: 5 white socks